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Heat transfer and fluid flow analysis Laminar Transport Phenomena Water and Water Ethylene Glycol solution from an Isothermal Sphere with Slip boundary conditions

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ABSTRACT: The laminar boundary layer flow and heat transfer of water and water Ethylene Glycol solution at identical combination from a permeable isothermal sphere in the attendance of thermal and hydrodynamic slip situation is analyzed. The surface of the sphere is maintained at a constant temperature. The transformed nondimensional equations are solved numerically with transformed boundary condition.The boundary layer conservation equations, which are parabolic in nature, are normalized into similar form and then solved numerically with the well-tested, efficient, MATLAB bvp4c code. The velocity slip and thermal are slips with constant non newtonian constant and the effect of prandtl number with temperature of the fluid considered for the present analysis. The obtained results are presented graphically.

KEYWORDS: Non Newtonian, fluid mechanics, Prandtl number, slip condition, thermal slip, heat transfer, skin friction, Nusselt number,

I. INTRODUCTION

Newtonian transportation phenomena occur in a lot of brushwood of development mechanical, chemical, and materials engineering. Such fluids exhibit shear-stress-strain relationships which diverge significantly from the classical Newtonian . Most non--Newtonian models involve some form of modification to the momentum conservation equations. These include power-law fluids[1], viscoplastic fluids counting Maxwell upper renewed models [2], Walters-B short memory models [3, 4], Oldroyd-B models [5], differential Reiner-Rivlin models [6,7], and Bingham plastics [8].The flow of non-Newtonian fluids in the incidence of heat transfer is an vital research area due to its application to the optimized dispensation of chocolate [9], toffee, and other foodstuffs [10]. This simple, yet elegant rheological replica was introduced originally [11] to simulate industrial inks. This replica [12] constitutes a plastic fluid replica which exhibits shear thinning characteristics, yield stress and high shear viscosity. Neofytou [17] studied computationally the flow characteristics of both power-law and Non Newtonian fluids in symmetric sudden expansions, showing that the critical generalised Reynolds number of transition from symmetry to asymmetry and subsequently the inverse dimensionless wall shear stress are linearly related to the dimensionless wall shear rate. Kandasamy et al. [18] studied numerically the thermal convection in concentric annuli using a non newtonian model. Mass transfer in a flowing through an annular geometry was examined by Nagarani et al. [19] who derived analytical solutions and also considered boundary absorption effects. Hemodynamic simulations of Non newtonian blood flow in complex arterial geometries were studied by Shaw et al. [20]. Attia and Sayed-Ahmed [21] studied the unsteady hydro magnetic Couette flow and heat transfer in a Non newtonian fluid using the Crank-Nicolson implicit method, showing that Non newtonian number controls strongly the velocity overshoot and has a significant effect on the time at which the overshoot arises. Hayat et al. [22] obtained homotopic solutions for stagnation-point flow and heat transfer of a Non newtonian fluid along a stretching surface, also considering viscous heating effects. Mustafa et al. [23] very recently analysed also with a homotopy method, the transient dissipative flow and heat transfer of a Non newtonian fluid over a moving flat plate with a parallel free stream, showing that surface shear stress and surface heat transfer are increased with the Non newtonian fluid parameter and also Eckert number (viscous heating parameter). The studies invariably assumed the “no-slip” condition at the boundary. Slip effects have however shown to be significant in certain industrial thermal problems and manufacturing fluid dynamics systems. Sparrow and Lin [24] presented the first significant investigation of laminar slip-flow heat transfer for tubes with uniform heat flux. These studies generally indicated that velocity slip acts to enhance heat transfer whereas thermal slip (or “temperature jump”) depresses heat transfer. Many studies have appeared in recent years considering both hydrodynamic and thermal slip effects. Interesting articles of relevance to process mechanical engineering include Larrode et al. [25] who studied thermal/velocity slip effects in conduit thermal convection, Spillane [26] who examined sheet processing boundary layer flows with slip boundary conditions and Crane and McVeigh [27] which studied slip hydrodynamics on a micro-scale cylindrical body. Further studies in the context of materials processing include Crane and McVeigh [28]. Studies of slip flows from curved bodies include Wang and Ng [29] which studied using asymptotic analysis the slip hydrodynamics from a stretching cylinder. Wang [30] has also examined stagnation slip flow and heat transfer from an axially moving cylinder showing that heat transfer increases with slip, Prandtl number and Reynolds number, and that in the case of large slip, the flow field decays exponentially into potential flow. The objective of the present paper is to investigate the laminar boundary layer flow and heat transfer of a rheological fluid past an isothermal sphere. Mathematical modelling through equations of continuity and motion leads to dimensionless non-linear coupled differential boundary layer equations. The velocity and thermal slip conditions along with conservation law of mass, momentum and energy completes the problems formulation designed for velocity components and temperature. The present objective of the paper is analysing the water and water Ethylene Glycol solution mixed at 50:50 ratio. MATLAB code develop the to solve the altered differential equation. A stable values of non newtonian parameter, velocity slip, stagnation parameter and thermal slip and different Prandtl number for water at 30^{0}C, 40^{0}C, 50^{0}C, 60^{0}C for water and Ethylene Glycol solutions. The results are presented with graphically for dissimilarity of velocity, nondimensional temperature, Nusselt number and skin friction.

II. MATHEMATICAL MODELLING:

Consider the steady, laminar, 2-D, viscous, incompressible, buoyancy-driven convection heat transfer flow from an isothermal sphere embedded in a non-Newtonian fluid. The flow replica and physical co-ordinate system. Here x is measured along the surface of the sphere and y is measured normal to the surface, respectively, and r is the radial distance from symmetric axes to the surface. r = a sin(x/a), where a is the radius of the sphere. The gravitational acceleration, g acts downwards. Both the sphere and the fluid are maintained initially at the same temperature. Instantaneously they are raised to a temperature Tw > T∞, the ambient temperature of the fluid which remains unchanged. The rheological equation of state for an isotropic flow of fluid, following Steffe [10] in tensorial notation may be stated and Subbarao et.al [20] analysed the Casson Rheological fluid flow on isothermal sphere

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\sin\left(\frac{x}{a}\right) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

where u and v are the velocity components in the x- and y-directions, respectively, ,β thermal expansion coefficient,α the thermal diffusivity, and T the temperature. The boundary conditions are prescribed at the sphere surface and the edge of the boundary layer regime, respectively: The boundary conditions are prescribed at the sphere surface and the edge of the boundary layer regime, respectively,

$$\text{at } y=0\,, u=N_0\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y}\,, T=T_w+K_o\frac{\partial T}{\partial y}$$

$\text{at } y\to\infty\,, u\to 0, T\to T_\infty$ Where N0 and K0 is the velocity and thermal slip conditions .For N0= K0 =0 ,one of the recover the no slip boundary case. The stream function ψ is defined by

$$ru=\partial(r\psi)/\partial y \text{ and} \qquad (4)$$

$$rv=-\partial(r\psi)/\partial x \qquad (5)$$

therefore the continuity equation is satisfied. In order to write the governing equations and the boundary conditions in dimensionless form the following nondimensional parameters are introduced.

$$\eta=\frac{y}{a}(Gr)^{0.25}\ \ \xi=\frac{x}{a}\ \ f(\eta)=\frac{\psi}{\upsilon Gr^{0.25}}\,,\ Pr=\frac{\upsilon}{\alpha},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, G_r=\frac{g\beta(T_w-T_\infty)a^3}{\upsilon^2}.,\ \beta=\mu_b\frac{\sqrt{2}\pi_c}{\rho_y} \qquad (6)$$

Where ρ is the mass desnity and T_∞ is the ambient temperature

In view of eq. (6), eqs.(2)-(4) reduce to the following coupled, non-linear, dimensionless partial differential equations for momentum and energy for the regime:

$$\left(1+\frac{1}{\beta}\right)f''' + (1+\xi\cot\xi)ff'' - f'^2 = 0 \quad (7)$$

$$\frac{\theta}{Pr} + (1+\xi\cot\xi)f\theta' = 0 \quad (8)$$

The transformed boundary conditions are

$$\eta = 0, f = S, f' = S_f f''(0), \theta = 1 + S_T\theta'(0)$$
$$\eta \to \infty, f' = 0, \theta \to 0 \quad (9)$$

In these equations, the primes denote the differentiation with respect to η, $Pr = \nu/\alpha$ the Prandtl number, $S_f = N0Gr^{0.25}/a$ and $S_T = K0Gr^{0.25}/a$ are the non-dimensional velocity and thermal slip parameters respectively. The parameter suction or blowing at the wall is represented by $S = -(V_w a)/(\nu Gr^{0.25})$, when $f_w < 0$ for $V_w > 0$ for the case of blowing and $f_w > 0$ for $V_w < 0$ for the case of suction. For the solid sphere $f_w = 0$ .The skin-friction coefficient and Nusselt number, which are given by:

$$\frac{1}{2}C_f = \left(\frac{1}{Gr^3}\right)^{1/4} = f''(0) \text{ and}$$
$$Nu/Gr^{1/4} = -\theta'(0) \quad (10)$$

III. PROPERTIES OF WATER AND WATER ETHYLENE GLYCOL:

The water and Ethylene Glycol water are excellent cooling or heat agent in heat transfer. The properties of these fluid are reported as

Property	Water 30^0C	Water +Ethylene 50%+50% 30^0C
Density Kg/m^3	995.25	1068.9
Viscosity N-s/m^2	0.00179	0.000503
Thermal conductivity W/m K	0.6	0.4042
Heat capacity KJ/Kg K	4179.4	3319.89
Prandtl	5.7257	20

IV. NUMERICAL PROCEDURE:

The governing equations are solved numerically with the transformed boundary conditions with MATLAB code bvp4c. The water and water ethylene glycol solution at 50% :50% at different temperatures 30^0C,40^0C, 50^0C,60^0C are considered for the present problem. The other parameter non newtonian parameter ,thermal slip, velocity slip, and suction or injection parameter considered constant for the present problem

The analysis is considered for the variation temperature, velocity, Nusselt number and skin friction factor with the variation of prandtl number examined.

V. RESULT AND DISCUSSIONS:

The figure 1.0 &2.0 represents the variation of velocity profile and secondary velocity with different values of prandtl number at β =0.5,ξ =0.5,Sf =0.5,St =0.5,Sw =0.5 are represented the velocity profiles. The velocity gradients are decreases with increasing in the prandtl number and temperature due to decreasing in the viscosity of the water. The figure 3.0 represents the variation of nondimensional

Fig 1.0 variation of velocity profile for water with different Temperatures at and Pr at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$

Fig 2.0 variation of velocity gradient for water with different temperatures at and Pr at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$

temperatures with variation of prandtl number and temperature The temperature profiles are increases with increasing the Pr values due to increasing the thermal conductivity and decreases density.

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Fig3.0:Effect of Prandtl number on nondimensional temperature with velocity slip and thermal slip for water at at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$

Fig4.0 Variation of Nusselt number for water with different Pr at temperatures for velocity slip and thermal slip at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$.

Fig 4.0 & Fig 5.0 represents the variation of Nusselt number and skin friction with variation of Pr and Temperature. The Nusselt number is increasing with increasing the temperatures. The skin friction coefficient are have significance effect with variation of Pr.

The figure 6.0 represents the variation of velocity and secondary velocity with water and water ethylene glycol mixture at 50:50 volume concentration. The thermal properties of water and ethylene glycol solution exhibit more Pr number and thermal conductivity is low and the effective thermal conductivity of mixture is at moderate values.

Fig5.0 Variation of skin friction for water with different Pr at temperatures for velocity slip and thermal slip at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$.

Fig 6.0 Variation of velocity profile for water and Water Ethylene Glycol solution at 50:50 with different Temperatures at and Pr at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$

The velocity profiles are increases with have no much significance effect on Pr. Fig 7.0 represents the secondary velocity profiles for water and water +EG mixture. The secondary variation velocity are have no significance effect with variation of Pr and Temperatures.

The figure 8.0 shows the non dimensional temperature with nondimensional distance. The temperature profiles are increases with increasing the temperature and prandtl number. For mixture of water and EG the temperature profiles are more due to moderate thermal properties. The figure 9.0 & 10.0 represents the variation of Nusselt numbers and skin friction factor with nondimensional distance. The Nusselt numbers are increases with increasing

Fig 7.0 variation of velocity gradient for water and water Ethylene Glycol solution at 50:50 with different temperatures at and Pr at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$

Fig 8.0 Effect of Prandtl number on nondimensional temperature with velocity slip and thermal slip for water and Water Ethylene Glycol solution at 50:50 at at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$

the Pr and for mixture of water + EG is more comparatively water dur to high viscosity and thermal conductivity.
Fig 10 represents the variation skin friction with nondimensional distance. The skin friction have no significance effect of on Pr numbers.

Fig9.0 Variation of Nusselt number for water and water Ethylene Glycol solution at 50:50 with different Pr at temperatures for velocity slip and thermal slip at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$.

Fig10.0 Variation of skin friction for water and water Ethylene Glycol solution at 50:50 with different Pr at temperatures for velocity slip and thermal slip at $\beta = 0.5, \xi = 0.5, S_f = 0.5, S_t = 0.5, S_w = 0.5$.

VI. CONCLUSIONS:

Numerical solutions have been presented for the transport phenomena combined heat and flow of rheological fluid external to an isothermal sphere, with suction/injection effects and velocity/thermal slip. The replicahas been developed to simulate foodstuff transport processes in industrial manufacturing operations. A robust, extensivelyvalidated, implicit finite difference numerical scheme has been implemented to solve the transformed, dimensionless velocity and thermal boundary layer equations, subject to physically realistic boundary conditions. The computations have shown that the velocity and temperature profiles and Nusselt numbers and skin friction are reported graphically. The velocity profiles are increases with decreases in the temperature and Pr and Temperatures profiles are increases with increasing the Pr and Nusselt numbers are increases with increasing the temperature and skin friction are have no much significance effect with variation Pr .The numerical results shows that the comparatively water and water +EG solution mixture of water and EG have improved values.

NOMENCLATURE :

a	radius of the sphere
Cf	skin friction coefficient
f	non-dimensional steam function
Gr	Grashof number
g	acceleration due to gravity
K0	thermal slip factor
N0	velocity slip factor
Nu	local Nusselt number
Pr	Prandtl number
r(x)	radial distance from symmetrical axis to surface of the sphere
s_f	non-dimensional velocity slip parameter
st	non-dimensional thermal slip parameter
T	temperature
u, v	non-dimensional velocity components along the x- and y-directions, respectively
V	the linear (translational) fluid velocityρ vector
x	stream wise co-ordinate
y	transverse co-ordinate

Greek symbols

α	thermal diffusivity
β	the non-Newtonian Non newtonian Parameter
η	the dimensionless radial co-ordinate
θ	non-dimensional temperature
μ	dynamic viscosity
n	kinematic viscosity
ξ	the dimensionless tangential co-ordinate
ρ	mass density
ψ	dimensionless stream function

Subscripts

w	conditions on the wall
∞	free stream condition

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